Quasinormal modes of a scalar perturbation coupling with Einstein's tensor in the warped AdS_3 black hole spacetime

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Abstract

We have studied the quasinormal modes of a massive scalar field coupling to Einstein's tensor in the spacelike stretched AdS_3 black hole spacetime. We find that both the right-moving and left-moving quasinormal frequencies depend not only on the warped parameter v of black hole, but also on the coupling between the scalar field and Einstein's tensor. Moreover, we also discuss the warped AdS/CFT correspondence from the quasinormal modes and probe the effects of the coupling on the left and right conformal weights h_L and h_R of the operators dual to the scalar field in the boundary.

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I. INTRODUCTION

Three-dimensional gravity has been investigated extensively during the past several decades because it certainly offers potential insights into quantum gravity. One of the interesting theories of gravity in three dimensions is the topological massive gravity (TMG), which is described by Einstein-Hilbert action with a negative cosmological constant $\Lambda = -\frac{1}{l^2}$ and with a parity-violating gravitational Chern-Simons (GCS) term [1, 2]

$$S = \frac{1}{2\kappa^2} \int d^3x \sqrt{-g}\left(R + \frac{2}{l^2}\right) + \frac{1}{4\kappa^2\mu} \int d^3x \sqrt{-g}\tilde{\epsilon}^{\lambda\mu\nu}\Gamma^\rho_{\lambda\sigma}\left[\partial_\mu\Gamma^\sigma_{\nu\rho} + \frac{2}{3}\Gamma^\sigma_{\mu\tau}\Gamma^\tau_{\nu\rho}\right], \tag{1}$$

where κ is a constant related to the three-dimensional Newton's constant G_N by $\kappa^2 = 8\pi G_N$, $\tilde{\epsilon}^{\lambda\mu\nu}$ is the Levi-Civita tensor defined by $\epsilon^{\lambda\mu\nu}/\sqrt{-g}$ with $\epsilon^{012} = 1$. The quantity $\mu \equiv 3v/l$ is the coupling constant of the Chern-Simons term, where v is a dimensionless coupling parameter. It is shown that for general μ or v the AdS_3 vacua are unstable due to the massive graviton with negative energy in the bulk [3]. However, at the special point $\mu l = 1$ or $v = 1/3$, it is found that there exist a stable AdS_3 vacua and its boundary CFT has a purely right-handed chirality with the central charges of $c_L = 0$ and $c_R = 3l/G$ [3–5]. This means that at this chiral point ($\mu l = 1$) the asymptotic symmetry group is generated by a single copy of the Virasoro algebra so that the TMG theory becomes chiral. On the other hand, it is found that the warped AdS_3 vacua for every v is a stable vacuum solution in TMG theory [6]. The geometry of the warped AdS_3 vacua can be looked as a fibration of the real line with a constant warp factor over AdS_2, which results in that the isometry group of the warped AdS_3 vacua is $SL(2, R) \times U(1)$ rather than $SL(2, R)_L \times SL(2, L)_R$ [6]. One of a warped AdS_3 vacua is the spacelike warped AdS_3 one with fiber coordinate u (or τ)

$$ds^2 = \frac{l^2}{v^2 + 3}\left[-\cosh^2\sigma d\tau^2 + d\sigma^2 + \frac{4v^2}{v^2 + 3}(du + \sinh\sigma d\tau)^2\right], \tag{2}$$

where $\{u, \tau, \sigma\} \in (-\infty, \infty)$. For $v^2 > 1$, the warp factor $\frac{4v^2}{v^2+3}$ is greater than unity so that the spacetime is a spacelike stretched AdS_3 space. If $v^2 < 1$, one can find the warp factor is smaller than unity and the spactime becomes the spacelike squashed AdS_3. As $v^2 = 1$, the warp factor is equal to one and there is no stretching.

The black hole solution asymptotic to spacelike stretched AdS_3 space has been constructed in [6]. It is found that such a warped AdS_3 black hole possesses spacelike $SL(2, R)_R \times U(1)_L$ isometries and does not suffer from naked closed timelike curves (CTC) [6–8]. The spacelike stretched warped AdS_3 black hole has been investigated extensively in recent years [8–23] . One of the main motivations is to probe the effects of GCS term on the physical properties of the warped black hole. Kim et $al.$ [9] studied the absorption cross section

of the spacelike stretched warped AdS_3 black hole and found that the absorption cross section is unexpectedly deformed by the GCS term. Kim [10] and Birmingham [11] investigated the effects of the parameter v on the thermodynamic stability and the statistical entropy of the black hole, respectively. The another main motivation is to check the conjecture of a warped AdS_3/CFT_2 correspondence [8] which indicates that there exist dual between the asymptotical warped AdS_3 gravity in the bulk and the two-dimensional conformal field theory in the boundary. Chen *et al.* [12, 13] investigated the quasinormal modes of the scalar, vector and Fermion perturbations in this black hole background and found [12] that the quasinormal frequencies are in good agreement with the prediction of the warped AdS_3/CFT_2 correspondence. This conjecture was also supported by the thermodynamic properties and the hidden conformal symmetry of the warped AdS_3 black hole [8, 14]. These results will excite more efforts be devoted to the study of the warped AdS_3/CFT_2 correspondence.

On the other hand, we know that the quasinormal modes depend not only on the parameter of the black hole, but also on the coupling between the perturbational fields and the background spacetime. The simplest perturbational field is scalar field, which associated with spin-0 particles in the quantum field theory. Theoretically, the general form of the action with more couplings between the scalar field and the spacetime curvature can be described by

$$S = \int d^4x \sqrt{-g}\left[f(\psi, R, R_{\mu\nu}R^{\mu\nu}, R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}) + K(\psi, \partial_\mu\psi\partial^\mu\psi, \nabla^2\psi, R^{\mu\nu}\partial_\mu\psi\partial_\nu\psi, \cdots) + V(\psi)\right], \quad (3)$$

where f and K are arbitrary functions of the corresponding variables. In general, the presence of these nonlinear couplings in the action results in that the motion equation for the scalar field is no longer a second-order differential equation in this case. Thus, it is very difficult for us to obtain the quasinormal frequencies for such a scalar perturbation in the black hole background because these nonlinear coupling equations can not be generally decoupled. However, Sushkov found [24] recently that the motion equation of the scalar field can be returned to second-order differential equation when it is kinetically coupled to the Einstein tensor, which means that the theory is a "good" dynamical theory from the point of view of physics. The new coupling term $G^{\mu\nu}\partial_\mu\psi\partial_\nu\psi$ can be looked as a coupling between the kinetic term of scalar field and the background spacetime. The applications of this new coupling form to the cosmology and to the black hole physics have been done extensively in [24–30]. It is found that the presence of this new coupling solves the problem of graceful exit from inflation in the early Universe [24–27], and changes the stability of the black hole and enhances Hawking radiation of the black hole [28–30]. The main purpose of this paper is to investigate probe the effect of this

new coupling on the properties of the quasinormal mode in the spacelike stretched AdS_3 black hole spacetime and to check whether it is consistent with the prediction of the warped AdS/CFT correspondence.

The plan of our paper is organized as follows: In Sec.II, we will review briefly the warped AdS_3 black hole spacetime. In Sec.III, we will study the quasinormal modes of a scalar perturbation coupling with Einstein's tensor spacelike stretched AdS_3 black hole spacetime. Our results indicate that the presence of such a coupling modifies the behavior of the quasinormal modes in the black hole and changes the left and right conformal weights h_L and h_R of the operators dual to the scalar field in the boundary. Finally, in the last section we will include our conclusions.

II.   THE WARPED AdS_3 BLACK HOLE

In this section, we review briefly the warped AdS_3 black hole spacetime in [6–8]. Varying the action (1) with respect to the metric, one can find that the bulk equation of motion can be described by

$$G_{\mu\nu} - \frac{1}{l^2}g_{\mu\nu} + \frac{l}{3v}C_{\mu\nu} = 0, \tag{4}$$

with the Einstein's tensor $G_{\mu\nu}$ and the Cotton tensor $C_{\mu\nu}$ which is defined by

$$C_{\mu\nu} = \epsilon^{\ \sigma\lambda\mu}\nabla_\lambda\left(R_{\sigma\nu} - \frac{1}{4}g_{\sigma\nu}R\right). \tag{5}$$

A interesting solution in which the Cotton tensor does not vanish is the spacelike stretched warped AdS_3 black hole, whose metric can be given in standard ADM form by [6–8]

$$ds^2 = -N(r)^2 dt^2 + R(r)^2[d\theta + N^\theta(r)dt]^2 + \frac{l^2 dr^2}{4R(r)^2 N(r)^2}, \tag{6}$$

where the functions in the metric are defined as

$$R(r)^2 = \frac{r}{4}\left[3(v^2 - 1)r + (v^2 + 3)(r_+ + r_-) - 4v\sqrt{r_+ r_-(v^2 + 3)}\right], \tag{7}$$

$$N(r)^2 = \frac{(v^2 + 3)(r - r_+)(r - r_-)}{4R(r)^2}, \tag{8}$$

$$N^\theta(r) = \frac{2vr - \sqrt{r_+ r_-(v^2 + 3)}}{2R(r)^2}. \tag{9}$$

The horizons are located at $r = r_+$ and $r = r_-$, where $1/g_{rr}$ as well as the determinant of the (t, θ) metric vanishes. The Hawking temperature of the black hole (6) is

$$\frac{1}{T_H} = \frac{4\pi vl}{v^2 + 3}\frac{T_L + T_R}{T_L}, \tag{10}$$

with

$$T_R = \frac{v^2 + 3}{8\pi v l}(r_+ - r_-), \tag{11}$$

$$T_L = \frac{v^2 + 3}{8\pi v l}\left[r_+ + r_- - \frac{\sqrt{(v^2 + 3)r_+ r_-}}{v}\right]. \tag{12}$$

For the case $v^2 > 1$, we have physical black holes if $r = r_+$ and $r = r_-$ are non-negative. Moreover, it is free from the closed timelike curves. When $v = 1$, there is no stretching and the above black hole becomes the usual BTZ black hole by a coordinate transformation

$$t \to \frac{\rho_+ - \rho_-}{l}t, \quad \theta \to \phi - \frac{1}{l}t, \quad r \to \frac{\rho^2}{\rho_+ - \rho_-}, \tag{13}$$

where

$$\rho_\pm = \sqrt{r_\pm}(\sqrt{r_+} - \sqrt{r_-}). \tag{14}$$

If $v^2 < 1$, we will find that the closed timelike curves (CTC) always appear at large values of r when θ is identified. In this paper, we will focus only on the case $v^2 \geq 1$ and study the properties of quasinormal modes of the coupling scalar perturbations around these physical black holes.

III. QUASINORMAL MODES OF A SCALAR PERTURBATION COUPLING WITH EINSTEIN'S TENSOR IN THE WARPED AdS_3 BLACK HOLE SPACETIME

In order to study the quasinormal modes of a massive scalar field coupling to Einstein's tensor in a warped AdS_3 black hole spacetime, we must first obtain its wave equation in the background. The action S_m for the scalar field coupling to the Einstein's tensor $G^{\mu\nu}$ in the three-dimensional curved spacetime has a form [24],

$$S_m = \int d^3x\sqrt{-g}\left[-\frac{1}{2}\partial_\mu\psi\partial^\mu\psi - \frac{\eta}{2}G^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - \frac{1}{2}m^2\psi^2\right]. \tag{15}$$

The coupling between Einstein's tensor $G^{\mu\nu}$ and the scalar field ψ is represented by the term $\frac{\eta}{2}G^{\mu\nu}\partial_\mu\psi\partial_\nu\psi$, where η is coupling constant with dimensions of length-squared. This new coupling can be regarded as the interaction between the kinetic term of scalar field and the background spacetime.

A. The formulas for the quasinormal modes

Varying the action (15) with respect to ψ, one can find that the for the scalar field coupling to Einstein's tensor the Klein-Gordon equation is modified as[24, 29]

$$\frac{1}{\sqrt{-g}}\partial_\mu\left[\sqrt{-g}\left(g^{\mu\nu} + \eta G^{\mu\nu}\right)\partial_\nu\psi\right] - m^2\psi = 0. \tag{16}$$

The Einstein's tensor $G^{\mu\nu}$ for the metric (6) has a form

$$G^{\mu\nu} = \begin{pmatrix} Hg^{tt} & 0 & \frac{v^2}{l^2}g^{t\theta} \\ 0 & \frac{v^2}{l^2}g^{rr} & 0 \\ \frac{v^2}{l^2}g^{t\theta} & 0 & \frac{v^2}{l^2}g^{\theta\theta} \end{pmatrix}, \tag{17}$$

with

$$H = \frac{v^2}{l^2} + \frac{3(v^2-1)(v^2+3)(r-r_+)(r-r_-)}{l^2 r[3(v^2-1)r + (v^2+3)(r_+ + r_-) - 4v\sqrt{(v^2+3)r_+ r_-}]}. \tag{18}$$

Adopting to the ansatz

$$\psi = e^{-i\omega t + ik\theta}\phi(r), \tag{19}$$

and substituting Eqs.(17) and (19) into Eq.(16), we find that the radial equation for the scalar field coupling to Einstein's tensor in the black hole spacetime (6) reads

$$\frac{4}{l^2}R^2 N^2 \frac{d}{dr}\left(R^2 N^2 \frac{d\phi(r)}{dr}\right) + \left[\frac{l^2(1+\eta H)}{l^2 + \eta v^2}R^2\omega^2 + 2k(R^2 N^\theta)\omega + k^2 - \frac{l^2 m^2 N^2 R^2}{l^2 + \eta v^2}\right]\phi(r) = 0. \tag{20}$$

The coupling parameter η appears in the terms contained ω^2 and m^2 in above equation, which means that the coupling between the scalar field and Einstein's tensor will change the properties of the quasinormal frequencies in the warped AdS_3 black hole spacetime. Defining the variable

$$z = \frac{r - r_+}{r - r_-}, \tag{21}$$

we find that the radial equation (20) can be rewritten as

$$z(1-z)\frac{d^2\phi(z)}{dz^2} + (1-z)\frac{d\phi(z)}{dz} + \frac{1}{(v^2+3)^2}\left(\frac{A}{z} + B + \frac{C}{1-z}\right)\phi(z) = 0, \tag{22}$$

with

$$A = \frac{l^2}{(r_+ - r_-)^2}\left[2k + \omega\sqrt{r_+}(2v\sqrt{r_+} - \sqrt{v^2+3}\sqrt{r_-})\right]^2, \tag{23}$$

$$B = -\frac{l^2}{(r_+ - r_-)^2}\left[2k + \omega\sqrt{r_-}(2v\sqrt{r_-} - \sqrt{v^2+3}\sqrt{r_+})\right]^2, \tag{24}$$

$$C = \frac{l^2}{l^2 + \eta v^2}\left[3(v^2-1)[l^2 + \eta(2v^2+3)]\omega^2 - m^2 l^2(v^2+3)\right]. \tag{25}$$

According to the boundary condition of quasinormal modes that only ingoing mode exists near the horizon, we find that the asymptotic solution near $r \sim r_+$ (i.e., $z \sim 0$) has the form

$$\phi(z) = z^\alpha(1-z)^\beta F(a, b, c, z), \tag{26}$$

with

$$\alpha = -i\frac{\sqrt{A}}{v^2+3},$$

$$\beta = \frac{1}{2}\left(1-\sqrt{1-\frac{4C}{(v^2+3)^2}}\right),$$

where

$$c = 2\alpha+1, \tag{27}$$

$$a = \alpha+\beta+i\sqrt{-B}/(v^2+3), \tag{28}$$

$$b = \alpha+\beta-i\sqrt{-B}/(v^2+3). \tag{29}$$

Making use of the property of the hypergeometric function [31]

$$F(a,b,c;z) = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}F(a,b,a+b-c+1;1-z)$$

$$+ (1-z)^{c-a-b}\frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)}F(c-a,c-b,c-a-b+1;1-z), \tag{30}$$

one can obtain the asymptotic behavior of the wave function $\phi(z)$ at the spatial infinity (i.e., $z \to 1$)

$$\phi(z) \simeq z^\alpha(1-z)^\beta\frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}. \tag{31}$$

Since the effective potential in the radial equation tend to infinity as $r \to \infty$, here we must impose the physical requirement as in Refs.[12, 13] that the wavefunction is just purely outgoing at spatial infinity and its corresponding flux is finite in the warped AdS_3 black hole spacetime. After some careful analysis, it is found that all of the divergent terms in the flux are proportional to

$$\left|\frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}\right|^2. \tag{32}$$

Thus, the boundary condition that the flux at the asymptotic infinity is not divergent leads to

$$c-a=-n, \qquad \text{or} \qquad c-b=-n, \tag{33}$$

with n being a non-negative integer. These two relations could be also obtained by simply imposing vanishing Dirichlet condition at spatial infinity.

Combining Eqs.(27), (28) and the relation $c-a=-n$, we find the right-moving quasinormal frequency obeys to

$$-i\frac{l}{r_+-r_-}\frac{1}{v^2+3}(4k+\omega\delta)+\frac{1}{2}\left(1+\sqrt{1-\frac{4C}{(v^2+3)^2}}\right)=-n, \tag{34}$$

where

$$\delta \equiv 2v(r_+ + r_-) - 2\sqrt{(v^2 + 3)r_+ r_-}. \tag{35}$$

Solving Eq.(34), one can obtain the right-moving quasinormal frequency for the scalar field coupling to Einstein's tensor is

$$\omega_R = \frac{(v^2 + 3)(l^2 + \eta v^2)}{d^2 \delta^2 (l^2 + \eta v^2) - 3l^2(v^2 - 1)[l^2 + \eta(2v^2 + 3)]} \left[-d\delta \left(\frac{4kd}{v^2 + 3} + i(n + \frac{1}{2}) \right) - i(e + if) \right], \tag{36}$$

with

$$d \equiv \frac{l}{r_+ - r_-}. \tag{37}$$

The quantities e and f in Eq. (36) are defined by

$$e = \sqrt{\frac{\sqrt{E^2 + F^2} + E}{2}}, \quad f = \sqrt{\frac{\sqrt{E^2 + F^2} - E}{2}}, \tag{38}$$

with

$$
\begin{aligned}
E &= \left[\frac{1}{4} + \frac{m^2 l^4}{(l^2 + \eta v^2)(v^2 + 3)} \right] d^2 \delta^2 - \frac{3l^2(v^2 - 1)[l^2 + \eta(2v^2 + 3)]}{l^2 + \eta v^2} \times \\
&\quad \left[\left(\frac{1}{4} + \frac{m^2 l^4}{(l^2 + \eta v^2)(v^2 + 3)} \right) + (\frac{4kd}{v^2 + 3})^2 - (n + \frac{1}{2})^2 \right], \\
F &= -3(v^2 - 1)l^2(n + \frac{1}{2})[l^2 + \eta(2v^2 + 3)] \frac{8kd}{(l^2 + \eta v^2)(v^2 + 3)}.
\end{aligned}
$$

For the left-moving quasinormal modes, we have

$$\frac{1}{2} \left(1 + \sqrt{1 - \frac{4C}{(v^2 + 3)^2}} \right) - i\frac{2vl\omega}{v^2 + 3} = -n, \tag{39}$$

which gives the left-moving quasinormal frequency

$$\omega_L = -i\frac{(l^2 + \eta v^2)}{l[l^2 + \eta(3 - 2v^2)]} \left[(2n + 1)v + h \right], \tag{40}$$

with

$$h = \sqrt{3(n + \frac{1}{2})^2(v^2 - 1) \left[\frac{l^2 + \eta(2v^2 + 3)}{l^2 + \eta v^2} \right] + \left[\frac{1}{4} + \frac{m^2 l^4}{(l^2 + \eta v^2)(v^2 + 3)} \right] (v^2 + 3) \left[\frac{l^2 + \eta(3 - 2v^2)}{l^2 + \eta v^2} \right]}.$$

Obviously, both the right-moving and left-moving quasinormal frequencies depend not only on the parameter v^2, but also on the coupling between the scalar field and Einstein's tensor. As the coupling parameter η vanishes, one can the formula (36) and (40) of quasinormal frequencies is consistent with that of the scalar field without coupling to Einstein's tensor [12, 13].

B. Dependence of quasinormal modes on the parameter v^2

We are now in the position to study the effect of the parameter v^2 on quasinormal modes. Here we focus only on the fundamental quasinormal modes ($n = 0$) because in this case the perturbations live longer in the black hole spacetime.

In Figs. (1)-(4), we plot the change of ω_R and ω_L with v for different values of η. Fig.(1) tells us that the real part of ω_R increases monotonously with the increase of v. From Figs. (2)-(4), we find that the change of imaginary parts becomes more complex. For $\eta = 0$, we find that with the increase of v the imaginary part
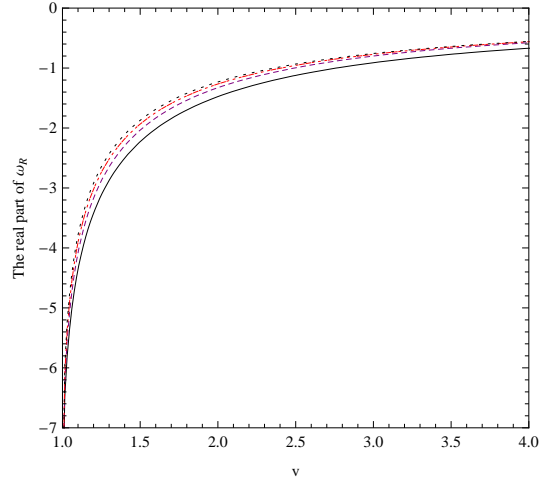

FIG. 1: Variety of the real part of the fundamental quasinormal modes ω_R with the parameters v and η. The solid, dashed, dash-dotted and dotted curves are for $\eta = 0$, 10, 20 and 30, respectively. Here, we set $r_+ = 1$, $r_- = 0.1$, $k = 2$, $L = 10$ and $m = 0.01$.
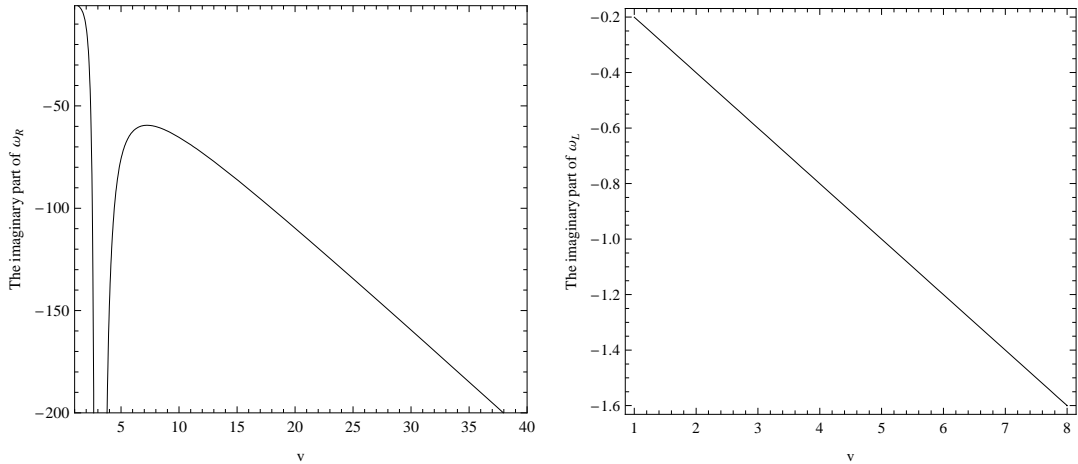

FIG. 2: Variety of the imaginary part of the fundamental quasinormal modes ω_R (the left) and ω_L (the right) with the parameters v for fixed $\eta = 0$. Here, we set $r_+ = 1$, $r_- = 0.1$, $k = 2$, $L = 10$ and $m = 0.01$.

of ω_L decreases. The imaginary part of ω_R decreases if $d^2\delta^2 - 3l^2(v^2 - 1) > 0$, but it first increases and then

decreases if $d^2\delta^2 - 3l^2(v^2 - 1) < 0$. Moreover, Figs.(1) and (2) tells us that both the imaginary parts of ω_L and ω_R are negative for arbitrary value of v, which means that the scalar perturbations always decay and the black hole is stable in this case. For $\eta \neq 0$, one can find that near $v \sim 1$ the imaginary part increases with v for ω_R and decreases for ω_L. When the parameter v is far from unit, it is easy to obtain that the
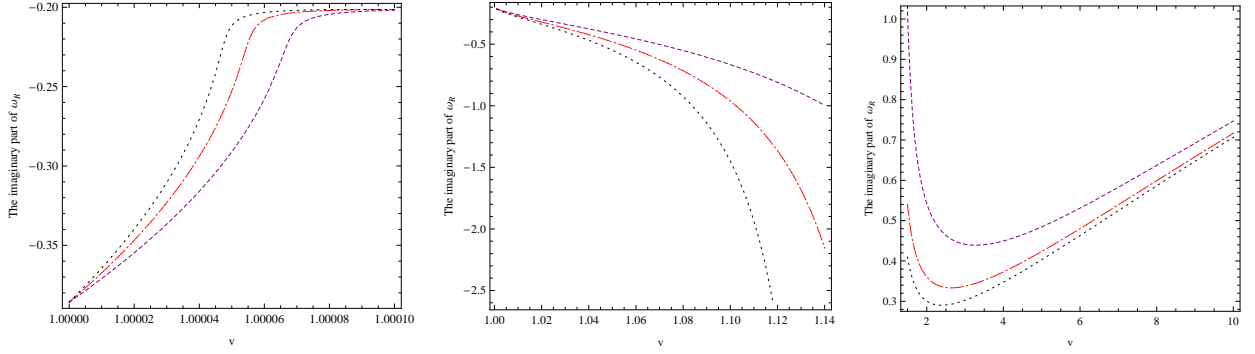

FIG. 3: Variety of the imaginary part of the fundamental quasinormal modes ω_R with the parameters v and η. The dashed, dash-dotted and dotted curves are for $\eta = 10$, 20 and 30, respectively. Here, we set $r_+ = 1$, $r_- = 0.1$, $k = 2$, $L = 10$ and $m = 0.01$.
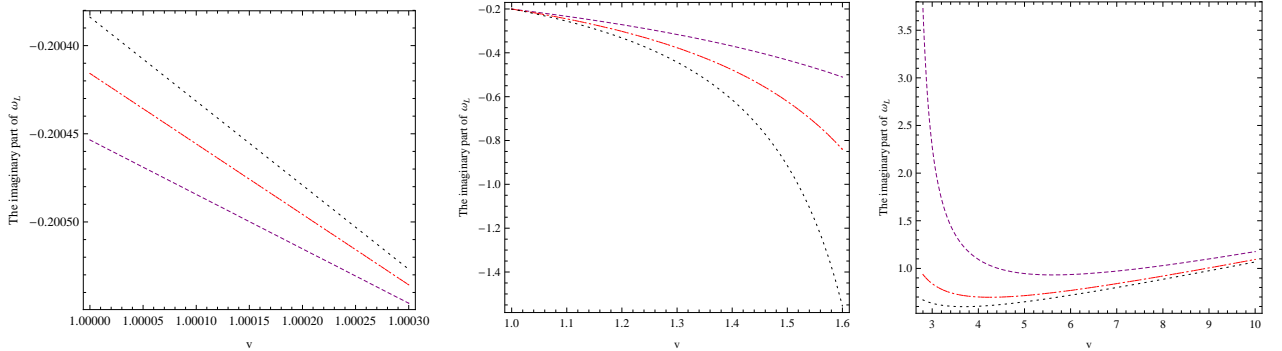

FIG. 4: Variety of the imaginary part of the fundamental quasinormal modes ω_L with the parameters v and η. The dashed, dash-dotted and dotted curves are for $\eta = 10$, 20 and 30, respectively. Here, we set $r_+ = 1$, $r_- = 0.1$, $k = 2$, $L = 10$ and $m = 0.01$.

imaginary part of ω_R is negative and it decreases with v as $d^2\delta^2(l^2 + \eta v^2) - 3l^2(v^2 - 1)[l^2 + \eta(2v^2 + 3)] > 0$. When $d^2\delta^2(l^2 + \eta v^2) - 3l^2(v^2 - 1)[l^2 + \eta(2v^2 + 3)] < 0$, we find that the imaginary part becomes positive and there exists a $U-$turn in the change of the imaginary frequencies. The presence of the positive imaginary part means that the scalar perturbations grow exponentially and then the background black hole is unstable in this case. This behavior of scalar quasinormal modes in the warped AdS_3 black hole spacetime is not found in the case without the coupling between the scalar perturbation and Einstein's tensor. For the left-moving modes ω_L, we find that the imaginary part for $v > 1$ have similar behavior except that the critical condition becomes $l^2 + \eta(3 - 2v^2) = 0$.

C. Dependence of quasinormal modes on the coupling parameter η

Comparing with Figs.(1)-(4), one can get the dependence of the quasinormal frequencies ω_R and ω_L on the coupling parameter η. It is shown in Fig.(1) that the real part of ω_R increases monotonously with the parameter v. Near $v \sim 1$, both the imaginary parts of ω_R and ω_L increase with η for fixed v. When the parameter v is larger than 1, both the imaginary parts decreases with η. Similarly, there also exists the growing modes for ω_R as $d^2\delta^2(l^2 + \eta v^2) - 3l^2(v^2 - 1)[l^2 + \eta(2v^2 + 3)] < 0$ and for ω_L as $l^2 + \eta(3 - 2v^2) < 0$. For bigger values of η, we find the the shape of $U-$ turn in the imaginary frequency become more sharp for ω_R and more flat for ω_L.

D. Quasinormal modes in the several limits

In this subsection we will discuss the properties of quasinormal modes in the several special case.

1. Case $v^2 = 1$

As $v^2 = 1$, the quasinormal frequencies (36) and (40) can be simplified as

$$\omega_R = -\frac{2k}{(\sqrt{r_+} - \sqrt{r_-})^2} - i\frac{2(\sqrt{r_+} + \sqrt{r_-})}{l(\sqrt{r_+} - \sqrt{r_-})}\left[n + \frac{1}{2}\left(1 + \sqrt{1 + \frac{m^2 l^4}{l^2 + \eta}}\right)\right], \tag{41}$$

$$\omega_L = -i\frac{2}{l}\left[n + \frac{1}{2}\left(1 + \sqrt{1 + \frac{m^2 l^4}{l^2 + \eta}}\right)\right]. \tag{42}$$

The real part of ω_R does not depend on the coupling parameter η, which is also shown in Fig. (1). The imaginary parts of both ω_R and ω_L are negative and increase with the coupling parameter η, which means that the black hole is stable and the scalar field coupling with Einstein's tensor decays more slowly in this case. According to AdS/CFT correspondence [32–35], the quasinormal frequencies (41) and (44) indicates that the presence of the coupling affects the left and right conformal weights of the operators dual to the scalar field in the boundary.

From the previous discussion, we know that as $v^2 = 1$ black hole becomes the usual BTZ black hole by a coordinate transformation (13). Thus, there exists a relation between the quasinormal frequencies of the BTZ black hole and the warped AdS_3 black hole with $v^2 = 1$

$$\omega_{BTZ} = \frac{\rho_+ - \rho_-}{l}\omega|_{v^2=1} + \frac{k}{l}. \tag{43}$$

Combining Eq. (14), (41) and (43), we can obtain the quasinormal frequencies of a scalar perturbation

coupling with Einstein's tensor in the BTZ black hole

$$\omega_R = -\frac{k}{l} - 2i\left(\frac{\rho_+ + \rho_-}{l^2}\right)\left[n + \frac{1}{2}\left(1 + \sqrt{1 + \frac{m^2 l^4}{l^2 + \eta}}\right)\right], \tag{44}$$

$$\omega_L = \frac{k}{l} - 2i\left(\frac{\rho_+ - \rho_-}{l^2}\right)\left[n + \frac{1}{2}\left(1 + \sqrt{1 + \frac{m^2 l^4}{l^2 + \eta}}\right)\right]. \tag{45}$$

As $\eta = 0$, the above result is consistent with the quasinormal frequencies of the scalar perturbations in the background of BTZ black hole [36, 37].

2. The extremal black hole case

For the extremal black hole with $r_+ \to r_-$, the quantity $d \to \infty$, which yields

$$\omega_R = -\frac{4k}{\delta}, \tag{46}$$

which is independent of the coupling parameter η. But for the left-moving one, the quasinormal frequencies is still a function of the coupling parameter η. Moreover, we also find that the frequencies in the extremal black hole background are pure real for the right quasinormal modes and are pure imaginary for the left one. From AdS/CFT correspondence [32–35], this implies that for the extremal black hole the right temperature is zero and the left temperature could be taken as the constant $\frac{v}{2\pi}$, which is coincident with that of the case $\eta = 0$ [12, 13].

3. Case $v \to \infty$

We now focus on an extreme case in which the black hole is stretched drastically so that the parameter $v \to \infty$. In this extreme stretching case, the expressions of the quasinormal frequencies can be expressed as

$$\omega_R = -i\frac{v}{l}\frac{\mathcal{T}}{\mathcal{T}^2 - 6}\left[n + \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{3}{2\mathcal{T}^2}\,n(n+1)}\right], \tag{47}$$

$$\omega_L = i\frac{v}{l}\left[n + \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{3}{2}n(n+1)}\right], \tag{48}$$

with

$$\mathcal{T} = \lim_{v \to \infty}\frac{T_L}{T_R} \tag{49}$$

As in the case without the coupling between the scalar field and Einstein's tensor, the real part of the quasinormal modes and the mass dependence are suppressed in the large v limit so that the quasinormal frequencies are pure imaginary and are proportional to v. Although the quasinormal frequencies do not

contain the coupling parameter η, one can find further that these quasinormal frequencies are different from those in the case $\eta = 0$ [12, 13], which tells us that the coupling modifies the behavior of the quasinormal modes in this limit. Moreover, we also find the imaginary part of ω_L becomes positive, which means that the left-moving modes makes the black hole unstable in this extreme stretching case.

E. The warped AdS/CFT correspondence and the quasinormal modes

Let us now to probe the relationship between the warped AdS/CFT correspondence and the quasinormal modes of the coupling scalar field in the warped AdS_3 black hole spacetime and probed the effects of the coupling on the left and right conformal weights. For the general case, it seems that the quasinormal frequencies of both the left-moving and right-moving modes are different from the prediction of AdS/CFT correspondence. However, making some proper local identification [12, 20],

$$t \to -\frac{v^2 + 3}{2}\tilde{\theta}, \quad \theta \to -\frac{v^2 + 3}{2v}\tilde{t}, \tag{50}$$

one can find that the quantum numbers in this two backgrounds satisfy the following relations

$$\tilde{\omega} = \frac{2}{v^2 + 3}k, \quad \tilde{k} = \frac{2v}{v^2 + 3}\omega. \tag{51}$$

Thus, the quasinormal frequencies (36) and (40) can be rewritten as

$$\tilde{\omega}_R = \frac{l}{v^2 + 3}\left[-4\pi T_L \tilde{k} - i4\pi T_R(n + h_R)\right], \tag{52}$$

$$\tilde{k} = -\frac{i}{l}(n + h_L), \tag{53}$$

respectively. Here, h_L and h_R are the conformal weight of the coupling scalar field with mass m, which can be expressed as

$$h_R = h_L = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{m^2 l^4}{(l^2 + \eta v^2)(v^2 + 3)} - \frac{3l^2(v^2 - 1)[l^2 + \eta(2v^2 + 3)]}{4v^2(l^2 + \eta v^2)}\tilde{k}^2}. \tag{54}$$

As in Ref. [12], the left temperature T_L in Eq. (53) could be recovered by defining the left-moving frequency in 2D CFT as $\tilde{\omega}_L = 2\pi T_L \tilde{k}$. Thus, from the quasinormal modes of the scalar field coupling to Einstein's tensor, one can find that the conjectured warped AdS/CFT correspondence could be still hold in the warped AdS_3 black hole spacetime. Moreover, we also find from Eq. (54) that with the increase of the coupling parameter η the conformal weights h_L and h_R decrease if the mass of the scalar field satisfies $m^2 > \frac{3(v^2 + 3)^2(v^2 - 1)}{4v^4}|\tilde{k}^2|$ and increase if $m^2 < \frac{3(v^2 + 3)^2(v^2 - 1)}{4v^4}|\tilde{k}^2|$.

IV. SUMMARY

In this paper, we studied the quasinormal modes of a massive scalar field coupling to Einstein's tensor in the spacelike stretched AdS_3 black hole spacetime. We find that the right-moving and left-moving quasinormal frequencies depend not only on the parameter v, but also on the coupling between the scalar field and Einstein's tensor. Near $v \sim 1$, the imaginary parts of both ω_R and ω_L increase with the coupling parameter η for fixed v, which means that the scalar field coupling with Einstein's tensor decays more slowly. However, for fixed η, with the increase of the parameter v, the imaginary part increase for ω_R and decreases for ω_L. For the larger values v, there exists the growing modes as $d^2 \delta^2 (l^2 + \eta v^2) - 3l^2 (v^2 - 1)[l^2 + \eta(2v^2 + 3)] < 0$ for ω_R and as $l^2 + \eta(3 - 2v^2) < 0$ for ω_L, which is not found in the case without the coupling between the scalar perturbation and Einstein's tensor. In the limit $v \to \infty$, the quasinormal frequencies become pure imaginary and do not contain the coupling parameter η, but they are different from those in the case $\eta = 0$. These results indicates that the coupling between the scalar field and Einstein's tensor modifies the behavior of the quasinormal modes in the spacelike stretched AdS_3 black hole spacetime. Moreover, we discussed the warped AdS/CFT correspondence from the quasinormal modes by making some proper local identification and probed the effects of the coupling on the left and right conformal weights of the operators dual to the scalar field in the boundary.

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